Study of the Polymer Flow Through Tubular Runner

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Flow through tubular runners presented is a very important flow type encountered in polymer processing, especially during injection mold filling. The filling pattern developed during the mold filling plays a decisive role on the final part quality, thus, in this paper we have investigated the filling pattern and the volumetric flow rate during the filling of a tubular runner for three different materials. The melt is treated as an incompressible, non-Newtonian the Ostwald-de Waele model with a power-law relationship between the shear stress and the shear rate.

Keywords: runner, injection molding, flow front, polymer melt

A major aim of the analysis of the injection molding process is to predict the way in which the mold is filled, i.e. the pattern of the advancing melt flow including melt front position. Also, another problem is related to the flow through the sprue and runner system of the molds. Suitable runner design can be decided on the basis of examination of the melt flow behavior within the runner system [1, 9]. On the other hand, the filling pattern developed during the mold filling plays a decisive role on the final part quality [8, 10].

The flow front is a moving free surface and the profiles of the melt front significantly vary along the filling path. Changes in direction of the flow or variations in flow rate reduce the part quality, thus the melt ideal flow should have a straight flow front across the mold, giving a uniform orientation pattern. In reality it is not straight and uniform, thus, preliminary studies of the flow behavior and the resulting trajectories of the melt flow are necessary to be carried out in order to asses the potential of the molding system. This can be done analytically or numerically using simulation software.

Numerical simulations are ideal suited to investigate the flow front behavior [7, 11]; however due to the complexity of the problem, the developed software are still too expensive and can not be used currently by the polymer processing industry.

In this paper the flow front profile including the penetration length and the volumetric flow rate for a mold cavity like straight circular tube are investigated using an analytical approach and validated using numerical simulation. Three different materials were considered: Low Density Polyethylene (LDPE), High Density Polyethylene (HDPE) and Polyvinyl chloride (PVC). The melt is treated as an incompressible, non-Newtonian the Ostwald-de Waele model with a power-law [5]. For analytical calculation, the melt is considered as isotherm and the pressure is constant, with varying flow rate [3, 5].

Mathematical model

In order to derive the analytical model, a straight cylindrical tube is considered, with geometry presented in figure 1, in which τ_{rz} is the shear stress, τ_R is the shear stress at wall and $v_z(r)$ is the flow velocity.

Using cylindrical coordinates (r, θ), assuming constant, steady-state Ostwald-de Waele fluid, and neglecting $\tau_{\theta\theta}$

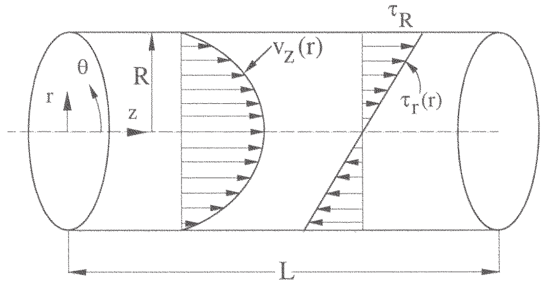

Fig. 1. Stress and velocity profile through the circular tube

and τ_{rr} in the *r*-momentum equation, the *z*-momentum equation is reduced to [2, 5, 6, 12]

$$\frac{1}{r}\frac{d}{dr}\left(r\,\tau_{rz}\right)=\frac{dp}{dz}. \tag{1}$$

The flow is unidirectional flow, $v_r = v_\theta = 0$, and thus there is only one non-zero velocity component, the *z* - component of velocity with radial position, $v_z = v_z(r)$, and $p = p(z)$

For a power law fluid, integrating both sides of the equation (1), the expression for shear stress is obtained as follows [9, 13]

$$\tau_{zr} = -m\left|\frac{dv_z}{dr}\right|^n, \tag{2}$$

in which m is the consistence index, and n is the rheological flow behavior index (n=1 corresponds to the Newtonian fluid).

The velocity gradient can be obtained from equation (2) and takes the form

$$\frac{dv_z}{dr}=-\left(-\frac{1}{2m}\frac{dp}{dz}\right)^{1/n} r^{1/n}. \tag{3}$$

The variation of the *z* - component of velocity with radial position is obtained by integrating the expression of the velocity gradient (3) as follows

$$v_z(r)=\left(\frac{n}{n+1}\right)\left[1-\left(\frac{r}{R}\right)^{(n+1)/n}\right]\left[-\frac{R^{n+1}}{2m}\frac{\Delta p}{dz}\right]^{1/n}. \tag{4}$$

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Since the velocity does not vary with z, the pressure gradient must be constant, therefore

$$\frac{dp}{dz} = \frac{\Delta p}{L}, \qquad (5)$$

in which Δp is the pressure drop along the tube, and L is the length of the cylindrical tube.

The volumetric flow rate is given by the equation

$$Q = \left(\frac{n\pi}{3n+1}\right)\left[-\frac{R^{3n+1}}{2m}\frac{\Delta p}{L}\right]^{1/n}. \qquad (6)$$

In order to investigate the flow front, the instantaneous flow rate $Q(t)$ at a constant inlet pressure P_o is considered. It can be expressed as follows

$$Q(t) = \frac{\pi R^3}{\varepsilon+1}\left(\frac{P_0}{2mZ(t)}\right)^{\varepsilon}, \qquad (7)$$

where $\varepsilon = 1/n$, and the melt front position for a time t is given by the equation

$$Z(t) = \frac{1}{\pi R^2}\int_0^t Q(t)dt. \qquad (8)$$

Using the boundary conditions: $Z(t) = 0$ at time $t = 0$ (at the initial time the mold cavity does not contain polymer melt), and $Z = Z(t)$ at a give time t, the equation (8) can be integrated as follows

$$Z(t) = \left(\frac{\varepsilon+1}{\varepsilon+3}\right)^{\frac{1}{\varepsilon+1}} R\left(\frac{P_0}{2m}\right)^{\frac{\varepsilon}{\varepsilon+1}} t^{\frac{1}{\varepsilon+1}}, \qquad (9)$$

and substituted into equation (7), to give the expression of the flow rate

$$Q(t) = \frac{\pi R^3}{\varepsilon+1}\left(\frac{\varepsilon+1}{\varepsilon+3}\right)^{\frac{1}{\varepsilon+1}}\left(\frac{P_0}{2m}\right)^{\frac{\varepsilon}{\varepsilon+1}} t^{-\frac{\varepsilon}{\varepsilon+1}}. \qquad (10)$$

The flow velocity can be also obtained as the first derivative of the equation (9) with respect to time

$$v_z(t) = \frac{dZ(t)}{dt} = \frac{Q(t)}{\pi R^2} \qquad (11)$$

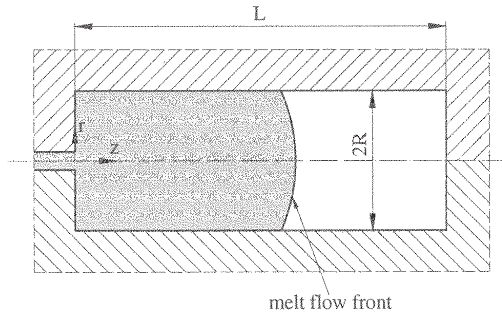

Fig. 2. Geometry of the tubular runner cavity

or, after substituting the expression of flow rate,

$$v_z(t) = \frac{R}{\varepsilon+1}\left(\frac{\varepsilon+1}{\varepsilon+3}\right)^{\frac{1}{\varepsilon+1}}\left(\frac{P_0}{2m}\right)^{\frac{\varepsilon}{\varepsilon+1}} t^{-\frac{\varepsilon}{\varepsilon+1}}. \qquad (12)$$

As can be seen in equation (9) the flow front position depends only on the radius R and the inlet pressure.

Numerical results

In order to determine the flow rate and the melt flow front position as a function of time, we have considered a straight tubular runner of length L = 300 mm and diameter d = 3 mm (fig. 2).

Three different materials were considered: LDPE (Low density polyethylene) Marlex® 1007 PE, HDPE (High density polyethylene) KS 10100 and PVC (Polyvinyl chloride) Vestolit Granulat 3411.

For comparison purpose, numerical simulations of the injection molding were carried out using the Moldflow Plastics Insight. The geometry of the injection mold is presented in figure 2. In order to characterize the rheological behaviour of the polymer melt, for each type of material, the coefficients for Cross-WLF model used in the Mold flow simulations were converted into the Power law model. The coefficients for the Cross-WLF model are given in table 1, while the flow index and the consistence index for the Power law model are presented in table 2. Figure 3 presents the variation of the viscosity for the Power model as a function of the shear rate. For the analytical calculation the time increment was set up to 0.1 seconds. The flow patters during the filling of the runner are presented in figure 4. The curvature of the flow front decreases as the distance

Table 1

CROSS - WLF VISCOSITY MODEL COEFFICIENTS FROM MOLDFLOW

Coefficient	Material		
	LDPE	HDPE	PVC
n	0.4278	0.321	0.4679
Ts (Pa)	22493	137000	7435.9
d1 (Pa Sec)	$5.60\cdot\times10^{11}$	2.01×10^{16}	$2.41\cdot\times 10^{23}$
d2 (K)	233.15	153.15	360.15
d3 (K/Pa)	0	0	0
a1	23.943	35.494	62.18
a2 (K)	51.6	51.6	51.6

Table 2

THE INJECTION PROCESSING PARAMETERS

Matreial	Flow index n	Consistency index m [$N\cdot s^n\cdot m^{-2}$]	Cavity fill pressure P_0 [$N\cdot m^{-2}$]	Cavity length L [m]	Cavity radius R [m]
LDPE	0.43	5575	$5\cdot\times10^6$	0.3	0.003
HDPE	0.33	22669	$12.5\cdot\times 10^6$		
PVC	0.46	38486	$30\cdot\times 10^6$		

Table 3
PREDICTED FILLING TIME

Material	Filling time [s]		
	LDPE	HDPE	PVC
Analytical calculation	5.00	6.90	8.40
Moldflow simulation	4.57	5.39	7.67

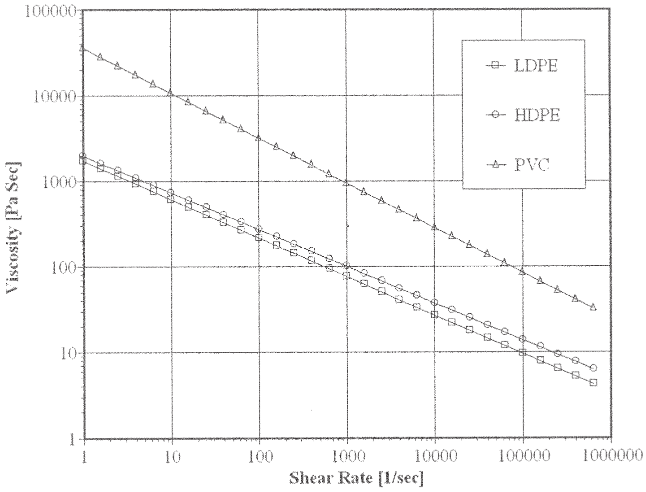

Fig. 3. Variation of viscosity

from the gate increases. The time required for the polymer melt to fill the cavity is given also in table. 3. The analytical results are compared in table 3 with the numerical solution.

The simulation results in terms of the filling time are shown in figures 5 – 7. Except for the HDPE, the analytical results agree fairly closely with the simulation results. The filling time computed using the analytical formulation is 10% higher that the filling time obtained using the Moldflow simulation, which is a good estimation taking into account the assumptions used in the analytical formulation. In case of HDPE, the results indicate that the filling time predicted by the analytical formulation is about 30% higher than obtained using the Moldflow simulation.

The time variation of the flow front position, i.e. penetration length $Z(t)$, is presented in figure 8, while the variation of the volumetric flow rate is depicted in figure 9. The variation of the flow velocity as a function of filling time is depicted in figure 10.

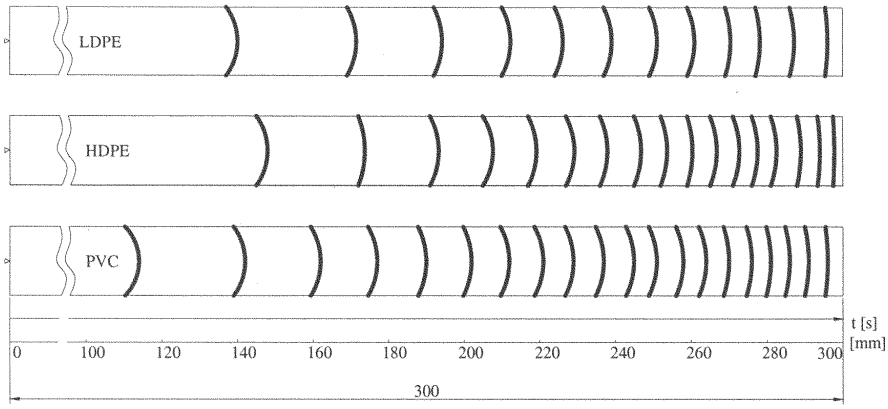

Fig. 4. Variation of melt front profile during the filling stage (time step of 0.4 s)

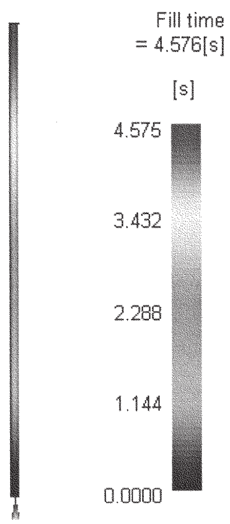

Fig. 5. Predicted fill time for LDPE using Moldflow simulation

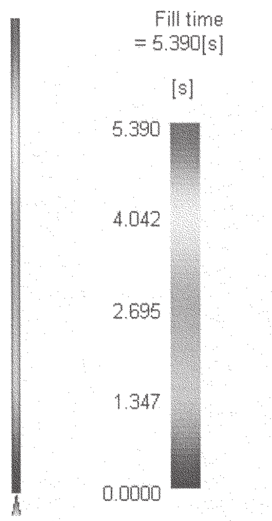

Fig. 6. Predicted filling time for HDPE using Moldflow simulation

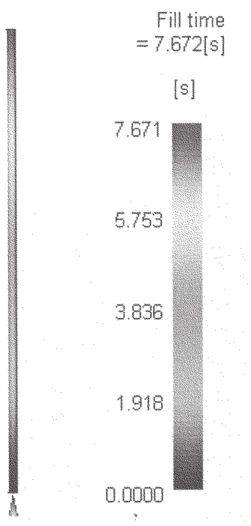

Fig. 7. Predicted filling time for PVC using Moldflow simulation

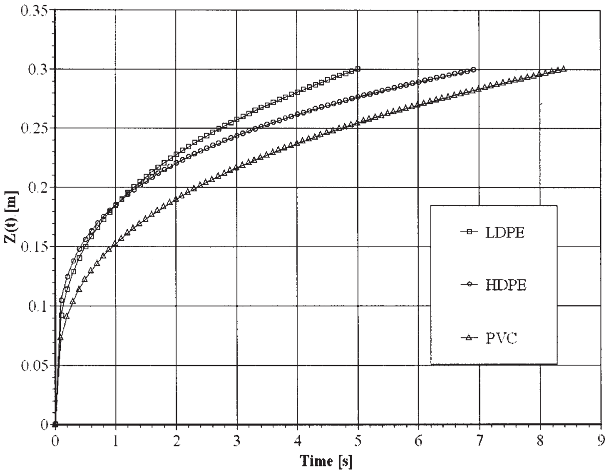

Fig. 8. Flow front position as a function of filling time

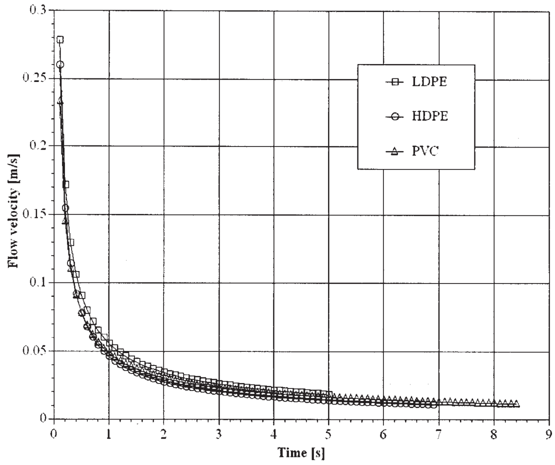

Fig. 10. Flow velocity as a function of filling time

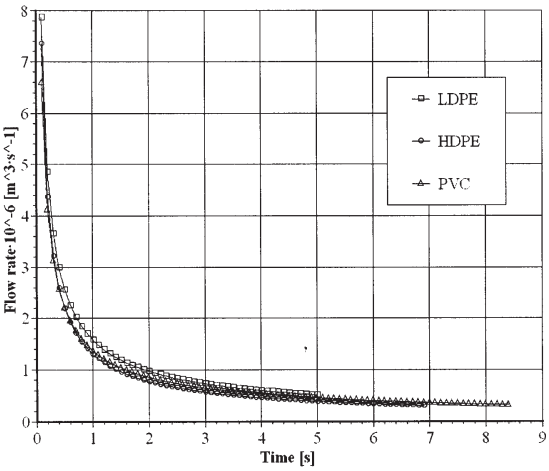

Fig. 9. Volumetric flow rate as a function of filling time

Conclusion

In this paper, the flow front profile including the penetration length, the flow rate and flow velocity for a mold cavity like circular tube were evaluated using an analytical approach and validated using the numerical simulation. It was found that a great deal of insight can be obtained by analyzing one-dimensional flow of polymer melt. The filling time and volumetric flow rate can be satisfactory described by using the analytical solution assuming isothermal flow.

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